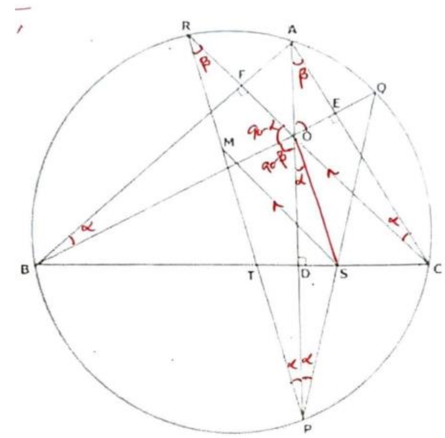
01.08.2023 – Cash Award Rider – Third Prize Winner Mr.Hara Gopal's Solution



Given : SM is parallel to CR

To Prove : RM = PT

Construction: Join OS

Proof:

As B, F, E, C are cyclic, $\angle ABE = \angle ACF = \alpha$ ------- (1)

As A, R, P, C are cyclic, $\angle ACR = \angle APR = \alpha$ ------- (2)

As A, F, E, C are cyclic, $\angle ABQ = \angle APQ = \alpha$ ------- (3)

From (1), (2) & (3) , $\angle ABQ = \angle ACR = \angle APR = \angle APQ = \alpha$

Let, $\angle PAC = \beta \Rightarrow \angle PRC = \beta$ (Same Segment Angles)

Here, we use a well known property of orthocenter, OD = DP ----------- (4)

In ΔODS & $\Delta\ PDS$,

OD = DP (from (4))

$\angle D = \angle D = 90°$

DS = DS (Common Side)
$\Delta ODS \cong \Delta PDS$ (SAS Congruency)

PS = SO -------------------------- (5)

In ΔPDS & $\Delta\ PDT$,

$\angle D = \angle D = 90°$

DP = DP (Common Side)
$\angle DPS = \angle DPT$ (proved)

$\Delta PDS \cong \Delta PDT$ (ASA Congruency)

PS=PT --------------------(6)

From (5) & (6)

PT = OS -------------------- (7)

Now, In ΔOSP, as OS = SP

$\Longrightarrow \angle SOP = \angle SPO = \alpha$ [Base angles of Iss.Δle]

In ΔAEO, as $\angle A = \beta \Longrightarrow \angle AOE = 90 - \beta$

$\Longrightarrow \angle BOD = 90 - \beta$ [V.O.A]

In ΔFBO, as $\angle B = \alpha \Longrightarrow \angle FOB = 90 - \alpha$

$\therefore \angle ROS = 90 - \alpha + 90 - \beta = 180 - \beta$ implies $\angle OSM = \beta$

According to angle sum property of Quadrilateral, In ROSM, $\angle RMS = 180 - \beta$

In the quadrilateral ROSM

$\angle R + \angle O = \beta + 180 - \beta$

Which means one pair & adjacent angles are supplementary and

RO∥ SM [As SM ∥ CR] & opposite angles are equal.

$\Longrightarrow$ ROSM is a parallelogram

And OS = RM and they are parallel

But OS=PT [Proved from (7)]

∴ RM = PT

----------------- Hence Proved.